EUROPEAN CYLINDER MAKERS ASSOCIATION

Development of calculation formulas for cylinder wall thickness

Technical Information: TI 002

Issue Date: 2018-01-02
Rev. No. 0

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1. Purpose

The purpose of this document is to explain the historical background and differences of the formulae used to calculate the minimum design wall thickness of seamless steel and aluminium alloy cylinders.

2. Scope

The scope of this document is limited to the following formulae which are used to calculate the minimum design wall thickness of seamless steel and aluminium alloy cylinders:

- Bach Clavarino formula
- Mean diameter formula
- Lamé von Mises formula

3. Introduction

Over the years three main formulae have been used to calculate the sidewall thickness of gas cylinders. These are:

a) the Bach Clavarino formula (e.g. used in DOT regulations)

$$S = \frac{[P_h(1{,}3D^2 + 0{,}4d^2)]}{D^2 - d^2}$$

$$a = \frac{D}{2} * \left(1 - \sqrt{\frac{S - 1{,}3 * P_h}{S + 0{,}4 * P_h}}\right)$$

b) the Mean Diameter formula (e.g. used in EEC Directives 84/525/EEC and 84/526/EEC, ISO4705 and many other national standards/codes/regulations in Europe)

$$a = \frac{P_h * D}{20 * R_{eg} * F + P_h}$$

c) the Lamé von Mises formula (e.g. used in new ISO designs)

$$a = \frac{D}{2} * \left(1 - \sqrt{\frac{10 * F * R_{eg} - \sqrt{3} * P_h}{10 * F * R_{eg}}}\right)$$

where the value of F is the lesser of $\frac{0{,}65}{R_{eg}/R_{mg}}$ or 0,85.

"Ref. ISO 9809-1"

Whereby the symbols have the following meaning:

a	calculated minimum wall thickness in mm (in inches for the Bach Clavarino formula)
D	outside diameter of the cylinder in mm (in inches for the Bach Clavarino formula)
d	inside diameter of the cylinder in inches
P_h	hydraulic test pressure in bar (in psi for the Bach Clavarino formula)
R_{eg}	minimum guaranteed value of yield stress in MPa
R_{mg}	minimum guaranteed value of tensile strength in MPa
F	design stress factor is the ratio of equivalent wall stress and test pressure to guaranteed minimum yield stress
S	wall stress in psi

4. Historical background and evaluation of design formulae by ISO/TC58/SC3

Many years ago, a Technical Committee of the International Standardisation Organisation ISO/TC58 was founded (1947) to develop international standards in the field of gas cylinders. This committee created a Sub-Committee ISO/TC58/SC3 which is responsible for the development of cylinder design standards.

The first ISO standard for seamless steel gas cylinders was published in 1983, namely ISO 4705. This standard used the Mean Diameter formula to calculate wall thickness. However, it was evident that two other formulae are also used worldwide to calculate wall thickness. Indeed, each formula results in slightly different wall thicknesses. As a consequence, ISO/TC58/SC3 established a working group WG-8 in 1981 (under the leadership of BSI) to investigate which of the three formulae most accurately reflects the actual stresses in the side walls of gas cylinders, see figure 1.

Cylinders with exactly defined wall thicknesses and mechanical properties, were produced and strain measurements were taken at increasing applied pressures up to and beyond test pressure to cause “elastic failure”.

The research project can be summarized as follows:
The general conclusion was that over the range of materials and cross sectional geometries investigated, the Lamé von Mises formulae were able to most closely predict elastic failure, and that this formula is therefore the preferred one for deriving required wall thickness of gas cylinders in terms of equivalent stress distribution.

The Bach Clavarino formula tends to overestimate internal pressure causing elastic failure, and the mean diameter formula tends to underestimate pressure to promote elastic failure for thin walls and overestimate for thick walls.

In more detail the outcome showed that gas cylinders can be categorized as:

- Thin-walled cylinders ($D_{external}$ / $D_{internal}$ around 1,06), are best calculated by the Lamé von Mises formula and is most applicable.
- Thick-walled cylinders with a diameter ratio $D_{external}$ / $D_{internal}$ around 1,16 – 1,20 like aluminium alloy or normalized mild steel cylinders, are best calculated by either the Mean Diameter or Lamé von Mises formula..
- Whereas, the Bach Clavarino formula results in slightly underestimated wall thicknesses (see graph below).

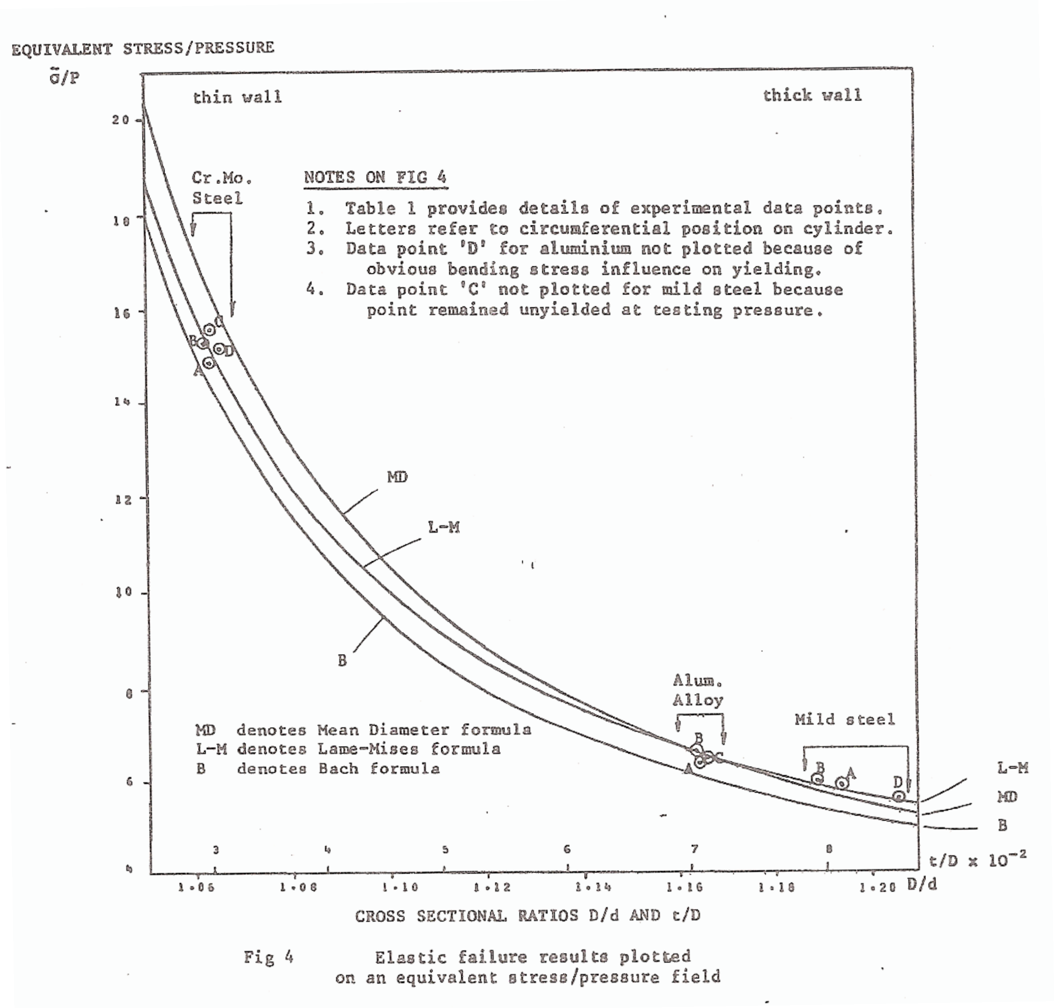

Figure 1: Elastic failure results plotted on an equivalent stress/pressure field, taken from "Paper regarding the test programme carried out by ISO TC58/SC3/WG-8"

Four examples of wall thickness calculations, using the three different design formulae, for cylinders having the same test pressure, diameter and mechanical properties can be found in Annex A to see the different influence on the minimum calculated wall thickness.

5. Conclusion

As a result of the studies by ISO/TC58/SC3/WG-8 the ISO committee agreed to use the Lamé von Mises formula in all future ISO cylinder design standards.

Industry experience over more than 15 years has shown the Lamé von Mises formula properly satisfies the wall thickness requirements for the design of seamless high pressure aluminium alloy and steel gas cylinders.

6. References

Paper regarding the test programme carried out by ISO TC58/SC3/WG-8:

Design Formulae for Transportable High Pressure Gas Cylinders
UK Test Programme for ISO/TC58/SC3/WG8
F McQuilken, Senior Scientific Officer, National Engineering Laboratory

Report from the research work carried out by ISO TC58/SC3/WG-8:

Design Wall Thickness of Transportable High Pressure Gas Cylinders
Comparison of Experiment and Theory
John A Walters, Bsc PhD, T.I. Chesterfield Limited

EEC Directive 84/525/EEC: Council Directive 84/525/EEC of 17 September 1984 on the approximation of the laws of the Member States relating to seamless, steel gas cylinders

ISO 4705: Refillable seamless steel gas cylinders

DOT 3AA, 49 CFR 178.37 - Specification 3AA and 3AAX seamless steel cylinders

ISO 9809-1, Gas cylinders - Refillable seamless steel gas cylinders - Design, construction and testing - Part 1: Quenched and tempered steel cylinders with tensile strength less than 1 100 MPa

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ANNEX A

Calculation examples for cylinders in accordance with the Bach Clavarino formula, the Mean Diameter formula and the Lamé von Mises formula

The following example calculations demonstrate the difference in in the calculated minimum wall thickness for steel cylinders made of the same material with the same mechanical properties (same heat treatment, quenched and tempered), the same test pressure and same diameter when calculated in accordance with (see table 1):

- the Bach Clavarino formula using the wall stress requirements of DOT 3AA (Example 1*)
- the Mean Diameter formula
  - using the F factor requirements of ISO4705 (Example 2.a)
  - using the F factor requirements of EEC Directive 84/525/EEC (Example 2.b)
- the Lamé von Mises formula using the F factor requirements of ISO 9809-1 (Example 3)

Specified parameters for calculation:

D = 229 mm (= 9,015748 inches)

P_h = 300 bar (= 4350 psi)

R_{mg} = 720,3477 MPa (= 104477,6 psi), see explanation in Example 1 below

Table 1: Summary of results of wall thickness calculations for gas cylinders with the same mechanical properties, test pressure and diameter for each of the three design formulae

Example no.	Design formula	Standard	Wall thickness in mm
1	Bach Clavarino formula	DOT 3AA (*Note that test pressure is 5/3 service pressure)	6,07
2a	Mean Diameter formula	ISO 4705	6,69
2b	Mean Diameter formula	EEC Directive 84/525/EEC	7,25
3	Lamé von Mises formula	ISO 9809-1	6,54

Example 1 – Bach Clavarino formula (DOT 3AA)

$$a = \frac{D}{2} * \left(1 - \sqrt{\frac{S - 1{,}3 * P_h}{S + 0{,}4 * P_h}}\right)$$

DOT3AA, §178.37 (f) (2) requires that the maximum specified wall stress S at test pressure shall not exceed 67% of the minimum tensile strength and must be not over 70000 psi. Therefore the following calculation is based on a specified minimum tensile strength of 70000/0,67=104477,6 psi (=720,3477 MPa).

$$a = \frac{9{,}015748}{2} * \left(1 - \sqrt{\frac{70000 - 1{,}3 * 4350}{70000 + 0{,}4 * 4350}}\right) = 0{,}2387167\ inches$$

a = 6,07 mm

Example 2.a - Mean Diameter formula (ISO4705)

$$a = \frac{P_h * D}{20 * R_{eg} * F + P_h}$$

The standard ISO4705 requires a yield/tensile strength ratio of max. 0,9, consequently the following minimum specified yield stress is taken: R_{eg} = 0,9 * R_{mg} = 648,3129 MPa

$$F = \frac{1}{1{,}3} = 0{,}76923$$

$$a = \frac{300 * 229}{20 * 648{,}3129 * 0{,}76923 + 300} = 6{,}687\ mm$$

a = 6,69 mm

Example 2.b - Mean Diameter formula (EEC Directive 84/525/EEC)

$$a = \frac{P_h * D}{20 * R_{eg} * F + P_h}$$

The EEC Directive requires a yield/tensile strength ratio of max. 0,85, consequently the following minimum specified yield stress is taken: R_{eg} = 0,85 * R_{mg} = 612,2955 MPa

$$F = \frac{3}{4} = 0{,}75$$

$$a = \frac{300 * 229}{20 * 612{,}2955 * 0{,}75 + 300} = 7{,}243\ mm$$

a = 7,25 mm

Example 3 - Lamé von Mises formula (ISO 9809-1)

$$a = \frac{D}{2} * \left(1 - \sqrt{\frac{10 * F * R_{eg} - \sqrt{3} * P_h}{10 * F * R_{eg}}}\right)$$

R_{eg} = 0,9 * R_{mg} = 648,3129 MPa

$$F = \frac{0{,}65}{R_{eg}/R_{mg}} = \frac{0{,}65}{648{,}3129\ /720{,}3477} = 0{,}722$$

$$a = \frac{229}{2} * \left(1 - \sqrt{\frac{10 * 0{,}722 * 648{,}3129 - \sqrt{3} * 300}{10 * 0{,}722 * 648{,}3129}}\right) = 6{,}54\ mm$$

a = 6,54 mm